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ENTROPY GENERATION ANALYSIS IN A VERTICAL POROUS CHANNEL WITH NAVIER SLIP IN THE PRESENCE OF VISCOUS DISSIPATION AND HEAT SOURCE

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ABSTRACT

The intension of this paper is to investigate the effects of Navier slip and buoyancy force on the entropy in a vertical porous channel with suction/injection. This problem is solved analytically by perturbation technique. Closed form solutions are obtained for the fluid velocity and the temperature. The leads of slip parameter injection/suction, Reynolds number, Peclet number, heat source parameter and Brinkman number on the fluid velocity, temperature profiles, and rate of entropy generation are showed graphically and quantitatively discussed.

***KEYWORDS:** Entropy Generation, Navier Slip, Porous Channel, Viscous Dissipation & Heat Source*

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INTRODUCTION

The scientific usage of the problems of irrigation, soil erosion and tile drainage are the present focus of the development of porous media flow [1–3]. Meanwhile, the problem of the slip flow regime is very important in this era of modern science, technology and vast ranging industrialization. In many practical applications, the fluid adjacent to a solid surface no longer takes the velocity of the surface. The fluid at the surface has a finite tangential velocity; it slips along the surface. The flow regime is called the slip flow regime and its effect cannot be neglected. The effects of slip conditions on the hydromagnetic steady flow in a channel with permeable boundaries were discussed by Makinde and Osalusi [4]. Khalid and Vafai [5] obtained the closed form solutions for steady periodic and transient velocity field under slip condition. Watanebe *et al.* [6] studied the effect of Navier Slip on Newtonian fluids at solid boundary.

The foundation of knowledge of entropy generation goes back to Clausius and Kelvins studies on the irreversibility aspects of the second law of thermodynamics. However, the entropy generation resulting from temperature differences has remained untreated by classical thermodynamics. Since entropy generation is the measure of the destruction of available work of the system, the determination of the active factors motivating the entropy generation is important in upgrading the system performances. Analysis of Entropy generation in slip regime on thermal micro-Couette flows was reported by Chen and Tian [7]. Chauhan and Kumar [8] were presented the leads of slip conditions on entropy generation and forced convection in a circular channel occupied by a highly porous medium. Flow of entropy generation and thermal characteristics inside a porous channel with viscous dissipation was noticed by Mahmud and Fraser [9]. Heat transfer and entropy generation effects in a channel on a compressible fluid flow with porous medium were made by Chauhan and Kumar [10].

Moreover, Chauhan and Rastogi [11] were investigated the entropy generation and heat transfer in MHD flow past a stretching sheet through a porous medium. Chinyoka et al [12] have discussed the analysis of entropy on an unsteady MHD magnetic flow with buoyancy effects through a porous pipe. Hooman et. al [13] has analyzed entropy generation optimization and heat transfer of forced convection in a porous-saturated duct of rectangular cross-section. Makinde [14] was studied the analysis of second law for variable viscosity on MHD flow with Newtonian heating and thermal radiation. Chen [15, 16] was discussed the entropy generation of double-diffusive convection in the presence of rotation and in rectangular cavity. Santha Das and Rabindra Nath [17] were presented the Entropy generation in a porous channel due to MHD flow with Navier slip.

In this paper, the irreversibility of a porous channel under the influence of buoyancy force and velocity slip is studied analytically by using perturbation method. The solution of resulting momentum and energy equations are presented for representative values of various physical parameters characterizing the fluid convection processes.

FORMULATION OF THE PROBLEM

The steady laminar viscous incompressible boundary layer flow through a porous vertical plates separated by a distance h with suction at the right wall, injection at the left wall and non-uniform temperature under the combined effects of Navier slip and buoyancy forces are considered. It is shown in the Figure 1.

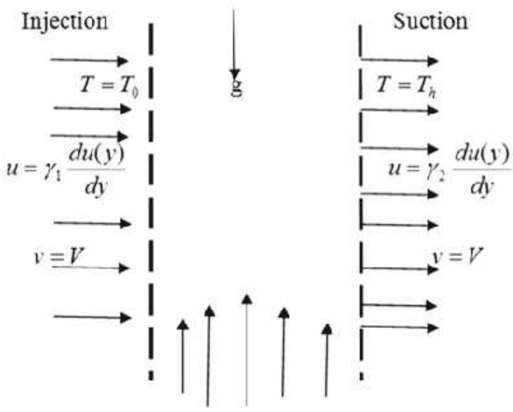

Figure 1: Physical Modal

The following assumptions are made:

- The y- axis is reserved as standard to the plate.
- The liquid flow is taken as x axis along the plate in the upward way.
- The movement is developed fully thermally and hydro dynamically.
- Heat generation/absorption and viscous dissipation effects are considered.
- The density difference is considered in the momentum equation.

By using Boussinesq's approximation due to effects of buoyancy. The energy and momentum equations for this flow are measured as:

$$V\frac{du}{dy}=-\frac{1}{\rho}\frac{dP}{dx}+\frac{\mu}{\rho}\frac{d^2u}{dy^2}+g\beta\left(T-T_0\right) \tag{1}$$

$$V\frac{dT}{dy}=\alpha\frac{d^2T}{dy^2}+\frac{\mu}{\rho c_p}\left(\frac{du}{dy}\right)^2+Q_1(T-T_0) \tag{2}$$

With the border conditions:

$$\left.\begin{array}{lll} v(0)=V, & u(0)=\gamma_1\dfrac{du(0)}{dy}, & T(0)=T_0 \\ v(h)=V, & u(h)=\gamma_2\dfrac{du(h)}{dy}, & T(h)=T_h \end{array}\right\} \tag{3}$$

where u is the fluid velocity, μ is the fluid viscosity, P is the pressure of the fluid, ρ is the fluid density, α is the thermal diffusivity, T is the temperature, c_p is the specific heat at constant pressure, β is volumetric expansion coefficient, γ_1 and γ_2 are slip coefficients, Q_1 is the generation of heat or constant of absorption and g is acceleration due to gravity.

We present the resulting non-dimensional variables:

$$\theta=\frac{T-T_0}{T_h-T_0},\quad K=-\frac{d\overline{P}}{d\overline{x}},\quad \overline{x}=\frac{x}{h},\quad \overline{P}=\frac{Ph}{\mu V},\quad W=\frac{u}{V},\quad \eta=\frac{y}{h},\quad \mathrm{Re}=\frac{V\rho h}{\mu},\quad Pe=\frac{Vh}{\alpha},$$

$$Br=\frac{V^2\mu}{\rho c_p\alpha\left(T_h-T_0\right)},\quad Gr=\frac{g\beta\rho h^2\left(T_h-T_0\right)}{\mu V},\quad \beta_1=\frac{\gamma_1}{h},\quad \beta_2=\frac{\gamma_2}{h},\quad Q=\frac{h^2Q_1}{\alpha}$$

where *Re* is the Reynolds number, *K* is the pressure gradient parameter, *Br* is the Brinkman number, Q is the heat source parameter, *Pe* is the Peclet number Gr is the Grash of number, β_1 and β_2 are the slip parameters.

On substituting these quantities of non-dimensional into equations (1)-(3), we get the equations of dimensionless and circumstances as follows:

$$\frac{d^2W}{d\eta^2}-\mathrm{Re}\frac{dW}{d\eta}+K+Gr\theta=0 \tag{4}$$

$$\frac{d^2\theta}{d\eta^2}-Pe\frac{d\theta}{d\eta}+Br\left(\frac{dW}{d\eta}\right)^2+Q\theta=0 \tag{5}$$

With the conditions

$$\left.\begin{aligned} W(0)=\beta_1\frac{dW(0)}{d\eta} \quad &, \quad \theta(0)=0 \\ W(1)=\beta_2\frac{dW(1)}{d\eta} \quad &, \quad \theta(1)=1 \end{aligned}\right\} \tag{6}$$

SOLUTION OF THE PROBLEM

With the conditions (6), the equations (4) and (5) are answered by using the perturbation technique:

$$W(\eta)=W_0(\eta)+BrW_1(\eta)$$

$$\theta(\eta)=\theta_0(\eta)+Br\theta_1(\eta)$$

In equations (4) - (6), on substituting these values and on equating the matching coefficients on both sides, we get the temperature and velocity equations as follows:

$$W_0^{11}-\mathrm{Re}W_0^1+Gr\,\theta_0+K=0 \tag{7}$$

$$W_1^{11}-\mathrm{Re}W_1^1+Gr\,\theta_1=0 \tag{8}$$

$$\theta_0^{11}-Pe\theta_0^1+Q\theta_0=0 \tag{9}$$

$$\theta_1^{11}-Pe\theta_1^1+Q\theta_1+W_0^{1^2}=0 \tag{10}$$

and the conditions are

$$\left.\begin{aligned} W_0(0)=\beta_1W_0^1(0) \quad , W_1(0)=\beta_1W_1^1(0) \quad , \quad \theta_0(0)=0 \quad , \quad \theta_1(0)=0 \\ W_0(1)=\beta_2W_0^1(1) \quad ,W_1(1)=\beta_2W_1^1(1) \quad , \quad \theta_0(1)=1 \quad , \quad \theta_1(1)=0 \end{aligned}\right\} \tag{11}$$

We get answers for velocity and temperature with the help of (7)-(11) as follows:

$$\theta=C_1\left(e^{m_1\eta}-e^{m_2\eta}\right)+Br\begin{pmatrix} C_7e^{m_1\eta}+C_8e^{m_2\eta}+C_9e^{2\mathrm{Re}\eta}+C_{10}e^{2m_1\eta}+C_{11}e^{2m_2\eta} \\ +C_{12}+C_{13}e^{(\mathrm{Re}+m_1)\eta}+C_{14}e^{(m_1+m_2)\eta}+C_{15}e^{m_2\eta}+C_{16}e^{\mathrm{Re}\eta} \\ +C_{17}e^{(\mathrm{Re}+m_2)\eta}+C_{18}e^{m_1\eta} \end{pmatrix} \tag{12}$$

$$\begin{aligned} W=&\left(C_2+C_3e^{\mathrm{Re}\eta}+C_4e^{m_1\eta}+C_5e^{m_2\eta}+C_6\eta\right) \\ &+Br\begin{pmatrix} C_{19}+C_{20}e^{\mathrm{Re}\eta}+C_{21}e^{m_1\eta}+C_{22}e^{m_2\eta}+C_{23}e^{2\mathrm{Re}\eta}+C_{24}e^{2m_1\eta}+C_{25}e^{2m_2\eta} \\ +C_{26}\eta+C_{27}e^{(\mathrm{Re}+m_1)\eta}+C_{28}e^{(m_1+m_2)\eta}+C_{29}\eta e^{\mathrm{Re}\eta}+C_{30}e^{(\mathrm{Re}+m_2)\eta} \end{pmatrix} \end{aligned} \tag{13}$$

ENTROPY GENERATION

Along a permeable conduit, the convection procedure is irreversible naturally. Exchange of energy and momentum causes the condition of non-equilibrium at the solid and inside the fluid borders. Thus, it is leading to the generation of continuous entropy in permeable channel. The generation of volumetric entropy in Cartesian coordinates as

$$E_G = \frac{\alpha}{T_0^2}\left(\left(\frac{dT}{dx}\right)^2 + \left(\frac{dT}{dy}\right)^2\right) + \frac{\mu}{\rho C_p T_0}\left[2\left\{\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dy}\right)^2\right\} + \left(\frac{du}{dy} + \frac{dv}{dx}\right)^2\right] \tag{14}$$

where the RHS first term of equation (14) is their reversibility of heat transfer and the second term is the generation of entropy due to the dissipation of viscous. The generation of entropy with the non- dimensional quantities decreases to

$$Ns = \frac{T_0^2 h^2 E_G}{\alpha(T_h - T_0)^2} = \left(\frac{d\theta}{d\eta}\right)^2 + \frac{Br}{\Omega}\left(\frac{dW}{d\eta}\right)^2 = N_1 + N_2 \tag{15}$$

Here, $\Omega = \dfrac{T_h - T_0}{T_0}$ is the parameter of temperature difference.

Here, N_1 gives irreversibility of heat transfer and N_2 represents the irreversibility of dissipative. To have an impression whether the irreversibility, due to the transfer of heat controls the entropy generation due to viscid overindulgence or vice–versa. Therefore, the irreversibility ratio is defined as

$$\varphi = \frac{N_2}{N_1} \tag{16}$$

If ϕ >1, then the generation of entropy due to viscous dissipation and the heat transfer irreversibility controls if 0< ϕ <1, but both equally contribute for ϕ = 1.

Hence, the Bejan number is well-defined as

$$Be = \frac{N_1}{N_S} = \frac{1}{1+\varphi} \tag{17}$$

Heat transfer irreversibility directs at the limit Be = 1, fluid friction irreversibility directs at the limit Be = 0 and the limit $Be = \frac{1}{2}$ indicates that both of them contribute equally.

RESULTS AND DISCUSSIONS

The rationality of borderline layer calculation for this canal flow issue can be qualified to the circumstance that the shared effects of suction and injection on this flow system are more marked within the walls of channel [9, 14]. The graphical representation and detailed analyzation of the results of above equation are offered in this segment. We take upright lines at η =0 as inoculation and at η =1 as force wall in this analyzation. We study the distance of non-

dimensional along X-axis and dimensionless speed along Y-axis. From figures 2-4, it is seen that there is a decrease in velocity at the suction wall and increase at the injection wall with the rise in Grash of number, the parameter of heat source and slip parameter β_1 and the other limitations are persistent. Maximum velocity is achieved by the flow near the central line. When the parameter of pressure gradient K, heat source parameter Q, Brinkman number Br, and Slip parameter β_2 increases, the profiles of temperature increases on both injection and suction channel walls, which are shown in figures 5-8. Figures 9-11 shows the entropy generation effect for several values of Br, Pe and β_2. It is recognized that the entropy generation decreases at injection wall and rises at the suction wall. It shows that there is less restrictive medium injection wall and more obstructive medium at the suction wall. Figure 12 displays the Brinkman number *Br* variations and its outcome on Bejan number. The graph shows that as Br increases, at the suction and injection channel walls, Bejan number is reducing. This indicates that due to the transfer of heat, the irreversibility decreases at both walls.

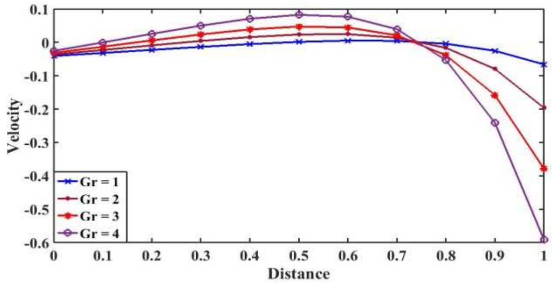

Figure 2: Velocity Effects for Gr with Re = 2, Br = 0.1, Q = 0. 1, Pe = 3, K = 0.1, $\beta_1 = 0.1$ and $\beta_2 = 0.1$

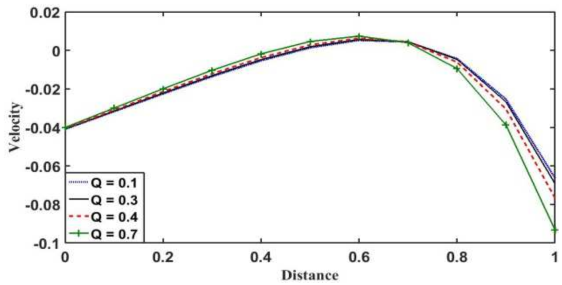

Figure 3: Velocity Profiles for Q with Gr = 1, K = 0.1, Br = 0.1, Re = 2, Pe = 3, $\beta_1 = 0.1$ and $\beta_2 = 0.1$

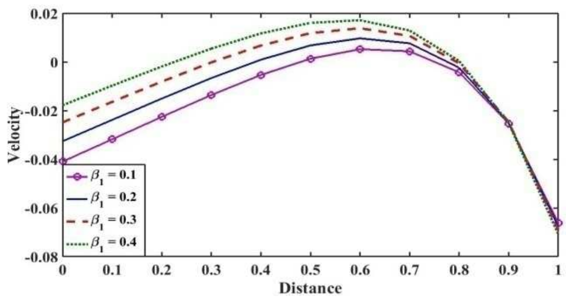

Figure 4: Velocity Profiles for β_1 with Gr = 1, K = 0.1, Br = 0.1, Re = 2, Q = 0.1, Pe = 3, and $\beta_2 = 0.1$

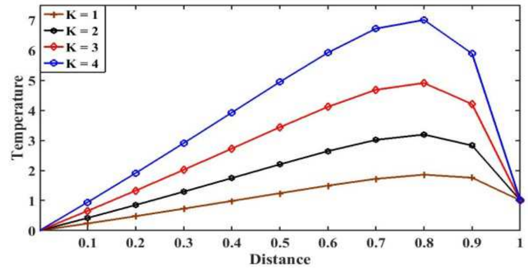

Figure 5: Temperature Effects for K with Gr = 1, Br = 0.1, Q = 1, Pe = 10, Re = 0.2, $\beta_1 = 0.1$ and $\beta_2 = 0.1$

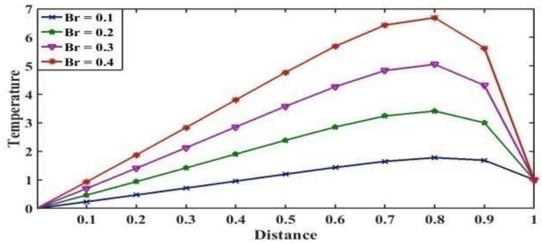

Figure 6: Temperature Effects for Br with Gr = 1, K = 3, Q = 1, Pe = 10, Re = 0.2, $\beta_1 = 0.1$ and $\beta_2 = 0.1$

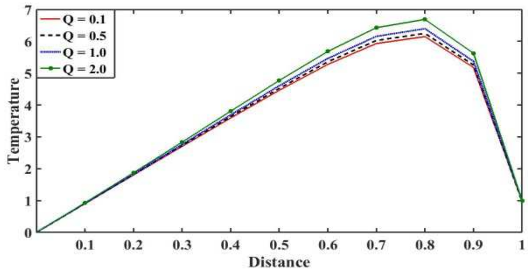

Figure 7: Temperature Effects for Q with Gr = 1, K = 3, Br = 0.1, Pe = 10, Re = 0.2, $\beta_1 = 0.1$ and $\beta_2 = 0.1$

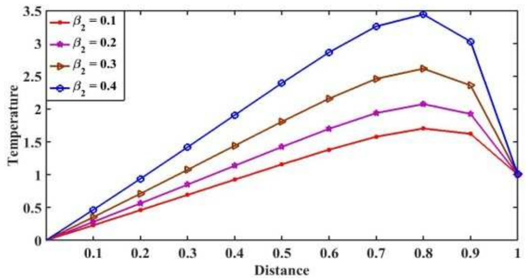

Figure 8: Temperature Effects for β_2 with Gr = 1, K = 3, Br = 0.1, Pe = 10, Re = 0.2, Q = 1 and $\beta_1 = 0.1$

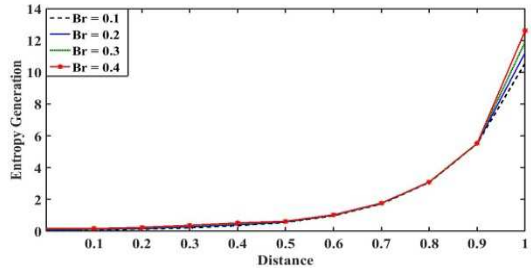

Figure 9: Entropy Generation for Br with Gr = 1, K = 0.1, Q = 0.1, Re = 2, Pe = 3, $\Omega = 1$, $\beta_1 = 0.1$ **and** $\beta_2 = 0.1$

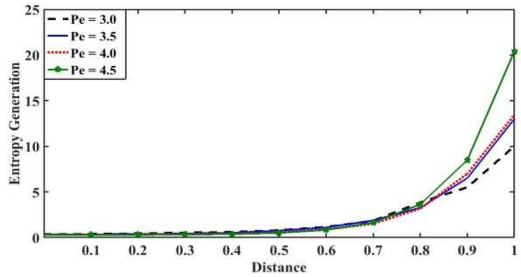

Figure 10: Entropy Generation for Pe with Gr = 1, Br = 0. 1, Q = 0.1, K = 0.1, Re = 2, $\Omega = 1$, $\beta_1 = 0.1$ **and** $\beta_2 = 0.1$

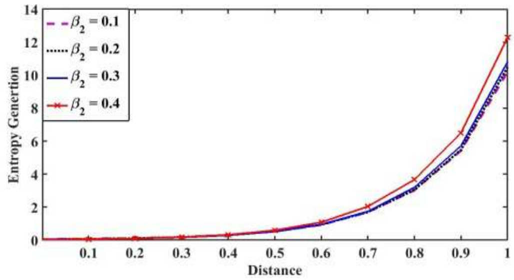

Figure 11: Entropy Generation for β_2 **with Gr = 1, Br = 0. 1, Q = 0.1, K = 0.1, Re = 2,** $\Omega = 1$, **Pe = 3 and** $\beta_1 = 0.1$

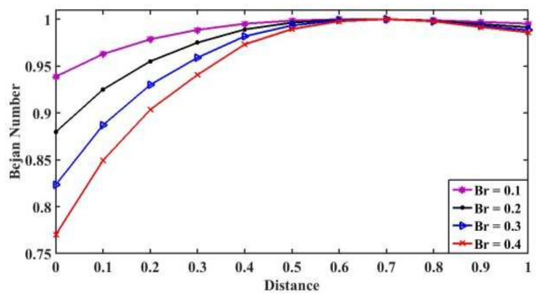

Figure 12: Effects of Bejan number for Br with Gr = 1, K = 0.1, Q = 0.1, Re = 2, Pe = 3, $\Omega = 1$, $\beta_1 = 0.1$ **and** $\beta_2 = 0.1$

CONCLUSIONS

The combined effect of buoyancy force and Navier slip on the generation rate of entropy through a vertical permeable conduit with suction/injection wall was examined. Velocity increases at the injection wall and decreases at the suction wall

with the increase in *Gr*, *Q* and β_1. When K, Br, *Q* and β_2 rises, the temperature profiles increases on both injection and suction channel walls. Entropy generation decreases at injection wall and increases at the suction wall due to Br, Pe and β_2. Bejan number declines at both suction and injection walls with the growing values of Br. These analytical outcomes attained by the means of perturbation method are in good terms with the outcomes done by the authors Ananthaswamy [18] by using Runge–Kutta–Fehlberg technique with shooting method by using homotopy perturbation technique and are also in good terms with the effects of the authors Eegunjobi [19].

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